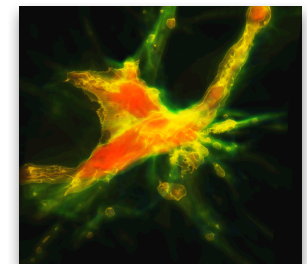
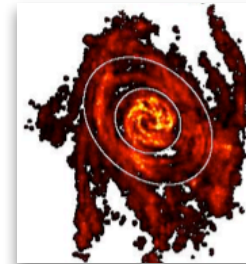
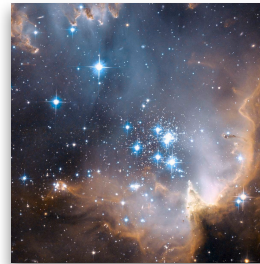
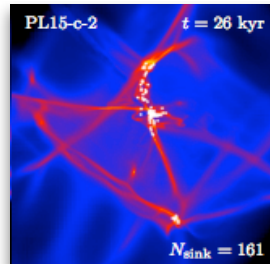
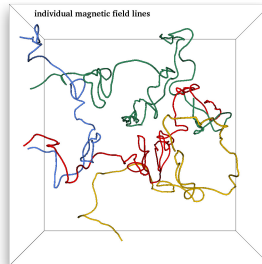


Modeling ISM Dynamics and Star Formation



Ralf Klessen



Universität Heidelberg, Zentrum für Astronomie
Institut für Theoretische Astrophysik



disclaimer

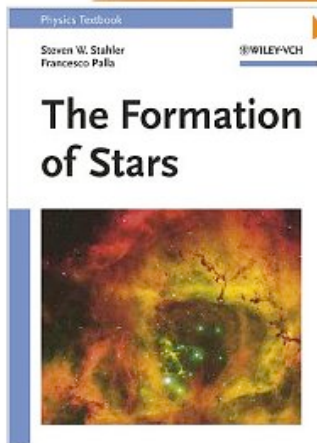
Disclaimer

- I try to cover the field as broadly as possible, however, there will clearly be a bias towards my personal interests and many examples will be from my own work.

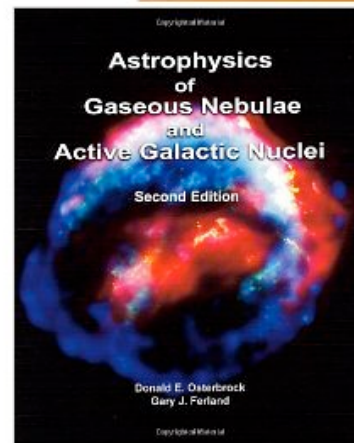
literature

Literature

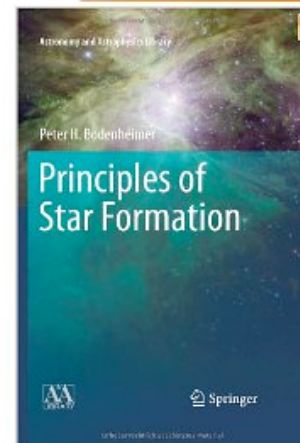
Click to **LOOK INSIDE!**



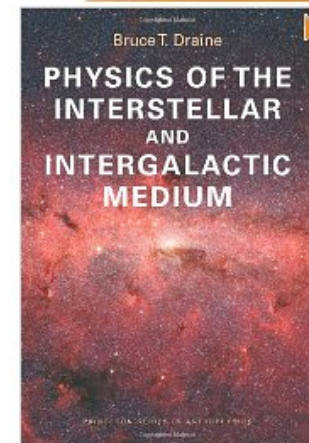
Click to **LOOK INSIDE!**



Click to **LOOK INSIDE!**



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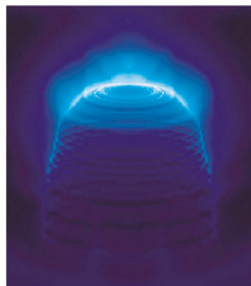


PHYSICS TEXTBOOK

George B. Rybicki
Alan P. Lightman

WILEY-VCH

**Radiative Processes
in Astrophysics**

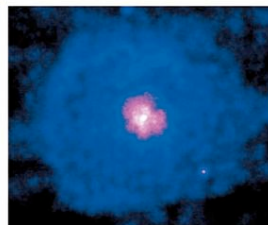


PHYSICS TEXTBOOK

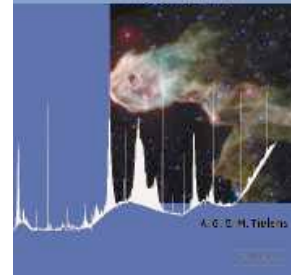
Lyman Spitzer, Jr.

WILEY-VCH

**Physical Processes in the
Interstellar Medium**



The Physics and Chemistry of the
**Interstellar
Medium**



Stars in Atmospheres and Atmospheres



**NUMERICAL METHODS
IN ASTROPHYSICS**

An Introduction

Peter Bodenheimer
Gerson P. Luger
Mohit Rastogi
Ramon N. Torricelli

Taylor & Francis

● Books

- Spitzer, L., 1978/2004, Physical Processes in the Interstellar Medium (Wiley-VCH)
- Rybicki, G.B., & Lightman, A.P., 1979/2004, Radiative Processes in Astrophysics (Wiley-VCH)
- Stahler, S., & Palla, F., 2004, "The Formation of Stars" (Weinheim: Wiley-VCH)
- Tielens, A.G.G.M., 2005, The Physics and Chemistry of the Interstellar Medium (Cambridge University Press)
- Osterbrock, D., & Ferland, G., 2006, "Astrophysics of Gaseous Nebulae & Active Galactic Nuclei, 2nd ed. (Sausalito: Univ. Science Books)
- Bodenheimer, P., et al., 2007, Numerical Methods in Astrophysics (Taylor & Francis)
- Draine, B. 2011, "Physics of the Interstellar and Intergalactic Medium" (Princeton Series in Astrophysics)
- Bodenheimer, P. 2012, "Principles of Star Formation" (Springer Verlag)



Literature

● Review Articles

- Mac Low, M.-M., Klessen, R.S., 2004, "The control of star formation by supersonic turbulence", Rev. Mod. Phys., 76, 125
- Elmegreen, B.G., Scalo, J., 2004, "Interstellar Turbulence 1", ARA&A, 42, 211
- Scalo, J., Elmegreen, B.G., 2004, "Interstellar Turbulence 2", ARA&A, 42, 275
- Bromm, V., Larson, R.B., 2004, "The first stars", ARA&A, 42, 79
- Zinnecker, H., Yorke, McKee, C.F., Ostriker, E.C., 2008, "Toward Understanding Massive Star Formation", ARA&A, 45, 481 - 563
- McKee, C.F., Ostriker, E.C., 2008, "Theory of Star Formation", ARA&A, 45, 565
- Kennicutt, R.C., Evans, N.J., 2012, "Star Formation in the Milky Way and Nearby Galaxies", ARA&A, 50, 531

Further resources

Internet resources

-  Cornelis Dullemond: *Radiative Transfer in Astrophysics*
http://www.ita.uni-heidelberg.de/~dullemond/lectures/radtrans_2012/index.shtml
-  Cornelis Dullemond: *RADMC-3D: A new multi-purpose radiative transfer tool*
<http://www.ita.uni-heidelberg.de/~dullemond/software/radmc-3d/index.shtml>

Part 2: Dynamics of the ISM

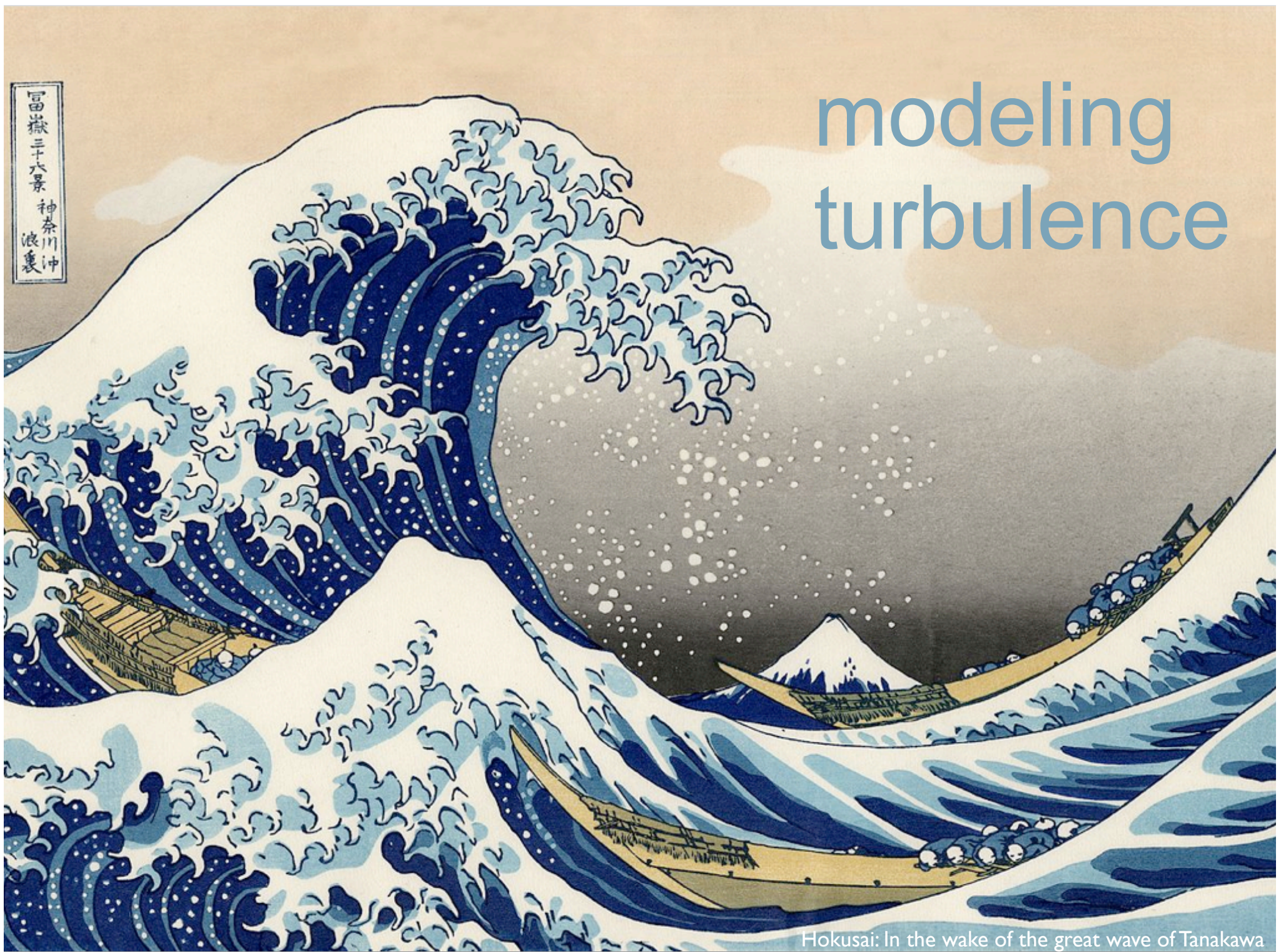


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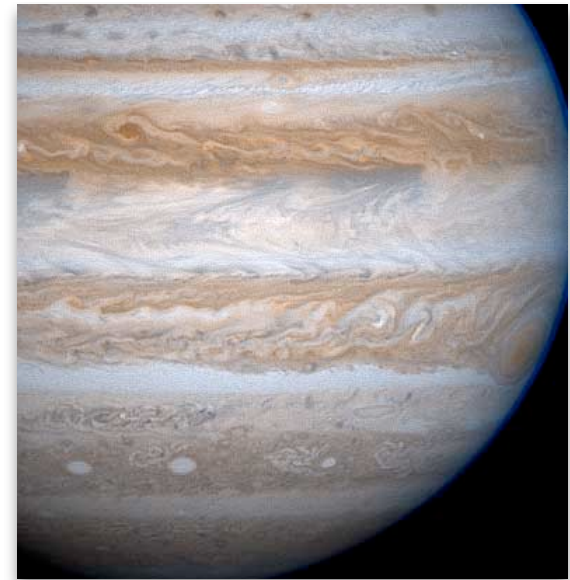
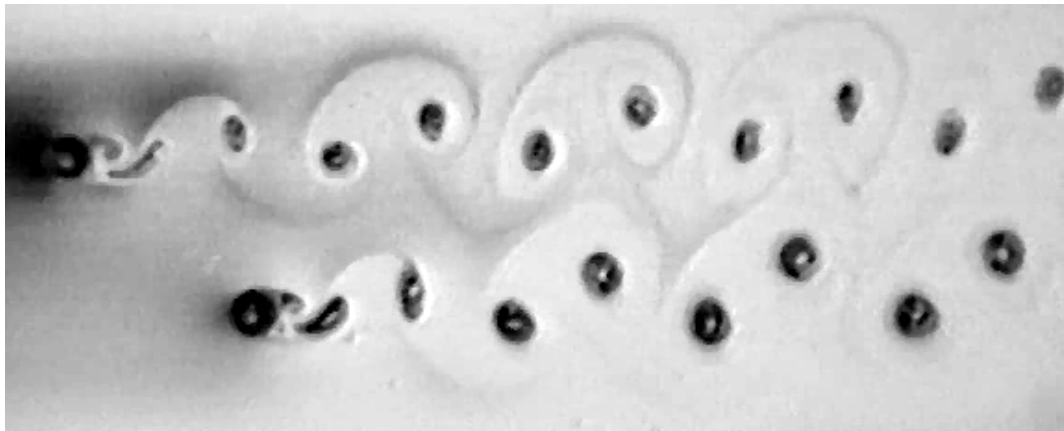


modeling turbulence



富嶽三十六景
神奈川沖
波裏

Hokusai: In the wake of the great wave of Tanakawa



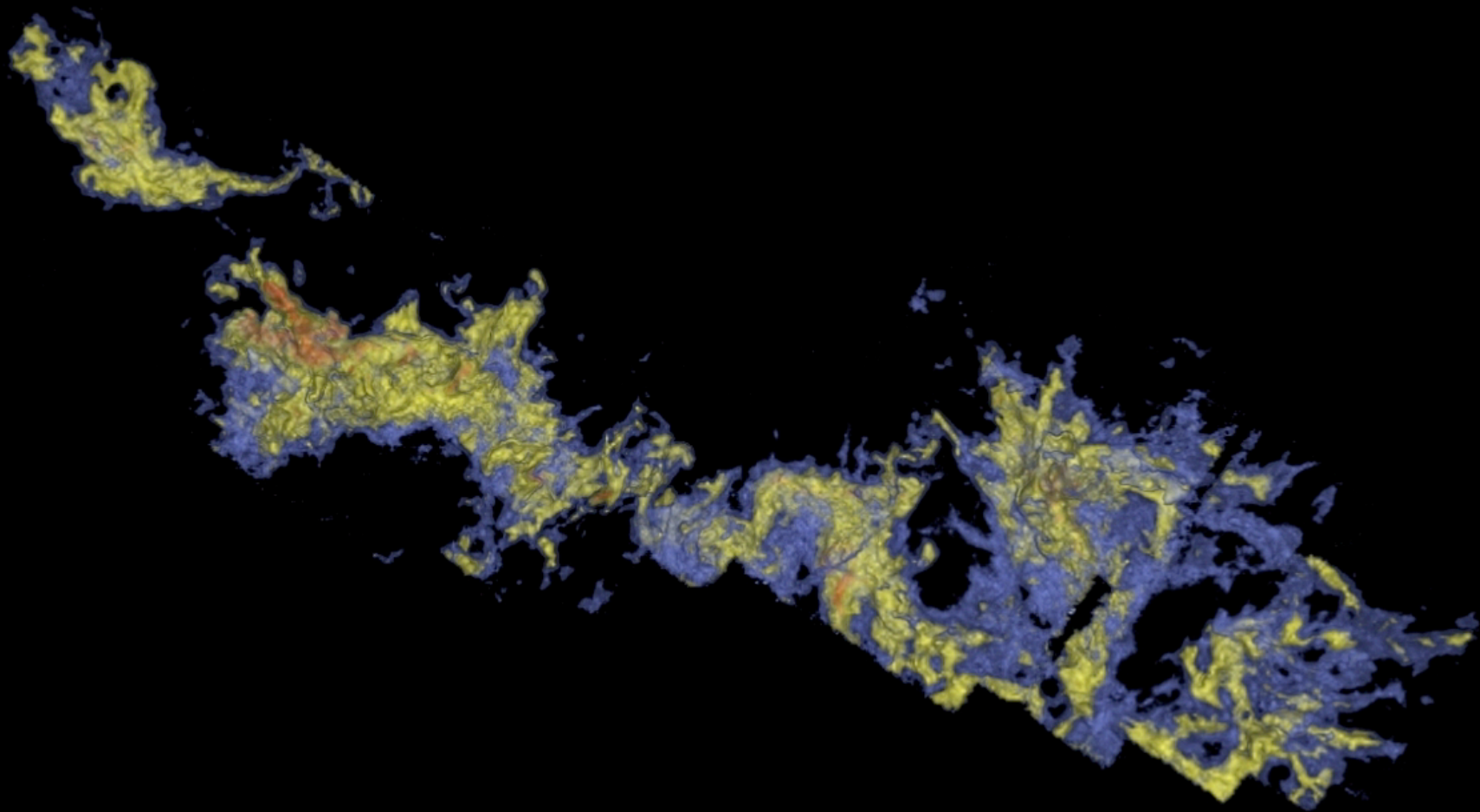
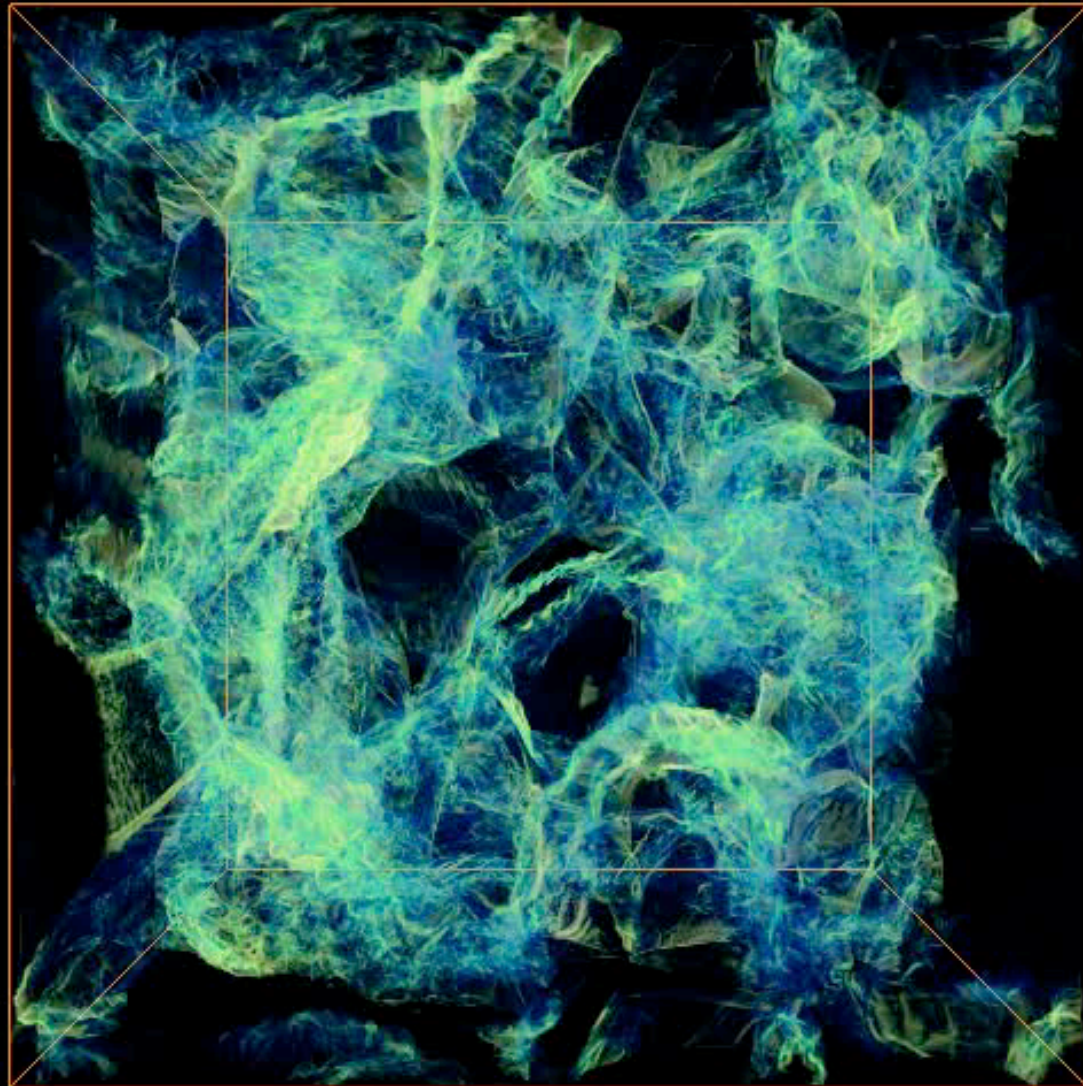


image from Alyssa Goodman: COMPLETE survey



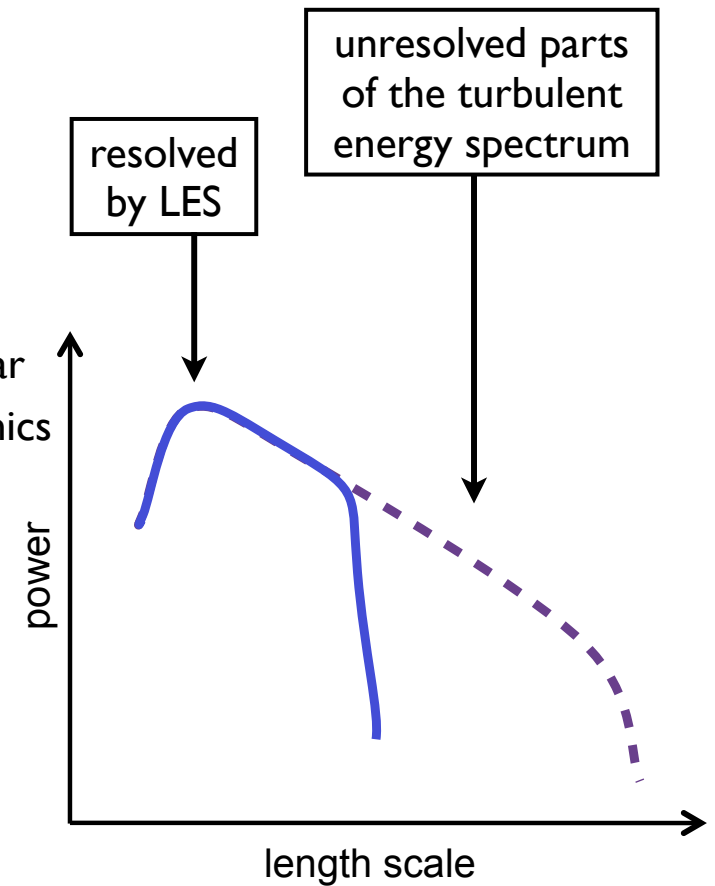
Schmidt et al. (2009, A&A, 494, 127)

large eddie simulations

- large eddie simulations (LES) attempt to resolve at least parts of the turbulent cascade
 - principal problem: only large scale flow properties
 - Reynolds number: $Re = LV/\nu$ ($Re_{\text{nature}} \gg Re_{\text{model}}$)
 - dynamic range much smaller than true physical one
- need subgrid model
 - (in our case simple: only dissipation)
 - more complex when processes (chemical reactions, nuclear burning, etc) on subgrid scale determine large-scale dynamics
- stochasticity → unpredictable when and where “interesting things” happen
 - occurrence of localized collapse
 - location and strength of shock fronts
 - etc.

large eddy simulations

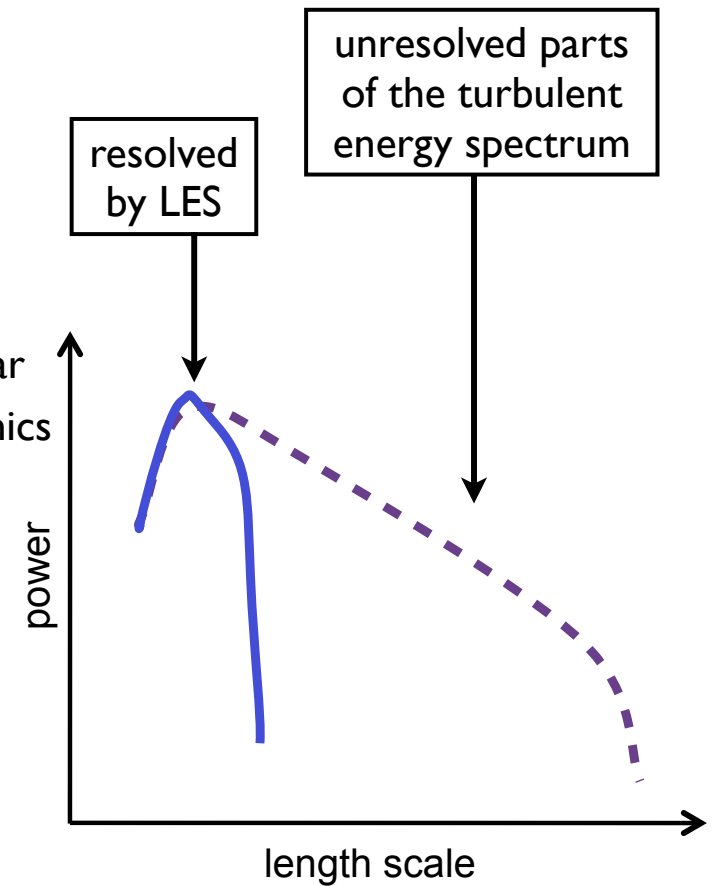
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We model honey instead of the ISM!!!



driving turbulence 0

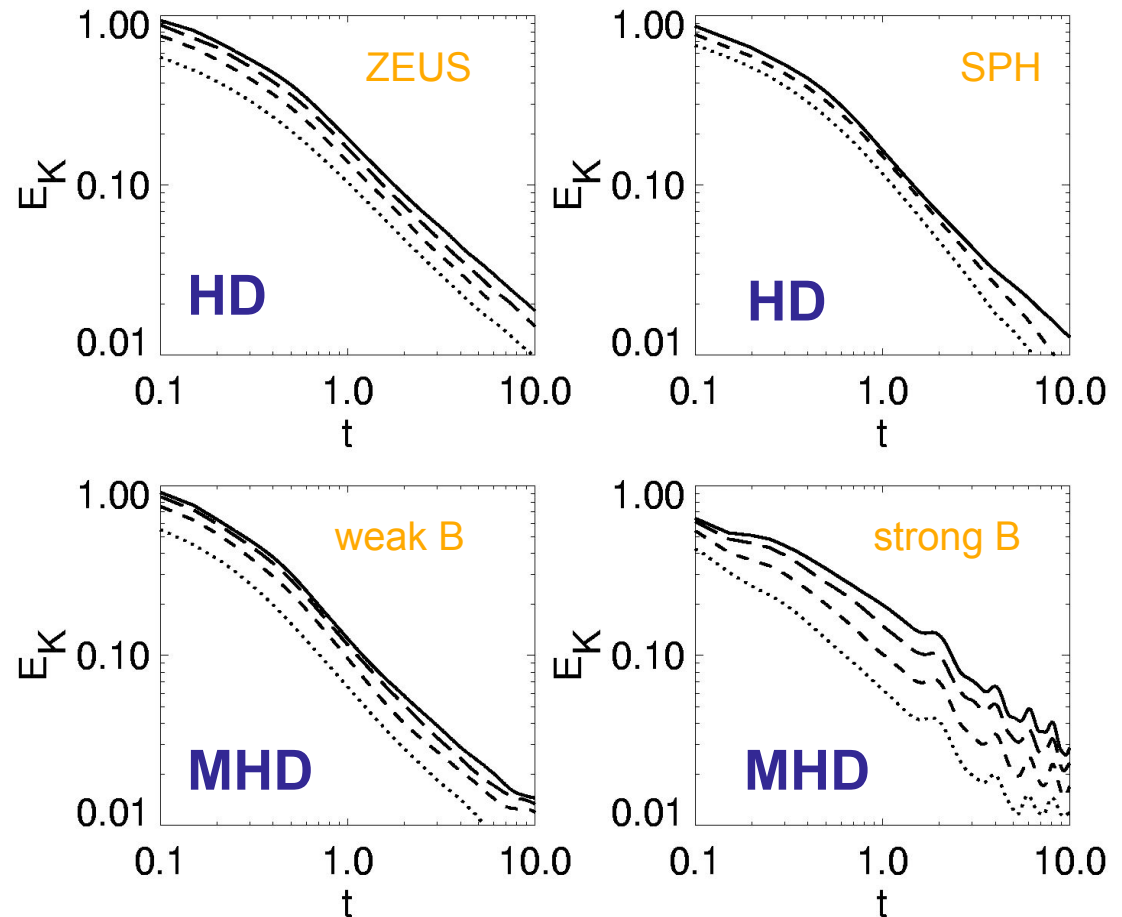
turbulence decays rapidly:

E_{kin} decays as $t^{-\eta}$, with $0.85 \approx \eta \approx 1.2$.

turbulence *decays* on timescales *comparable* to the free-fall time τ_{ff}

(e.g. Mac Low et al. 1998, Stone et al. 1998, Padoan & Nordlund 1999)

steady state turbulence needs to be continuously driven!



(Mac Low, Klessen, Burkert, & Smith, 1998, PRL)

driving turbulence 1

turbulent energy decays --> steady state turbulence needs to be driven --> insert energy at each timestep (or at least frequently)

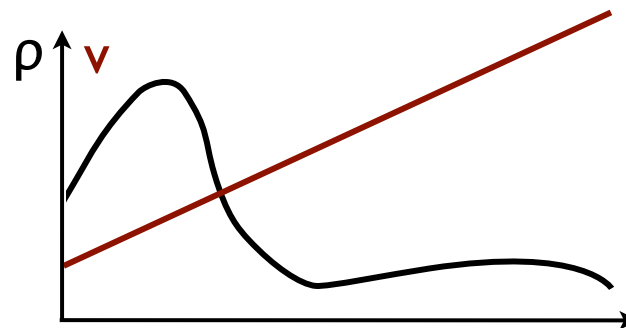
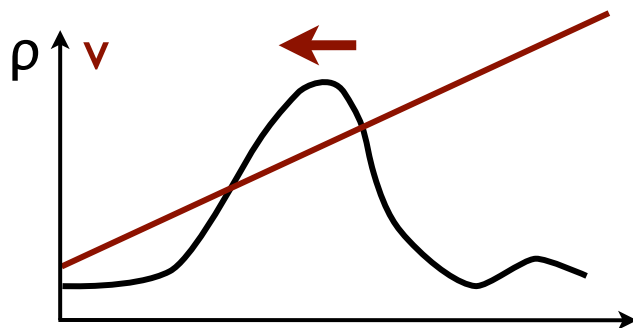
two possibilities:

-- include stochastic force term

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = \sum_i \vec{F}_i + \vec{f}_t$$

-- add $\delta \vec{v}_t$ to the velocity $\vec{v} \rightarrow \vec{v} + \delta \vec{v}_t$

for supersonic turbulence, keeping constant velocity dispersion requires some thoughts (because of compressibility of the medium)



driving turbulence 2

turbulent energy decays --> steady state turbulence needs to be driven
--> insert energy at each timestep (or at least frequently)

goal: keep rms velocity dispersion constant
--> adjust the amount of energy added

$$\Delta E = \sum_i \frac{m_i}{2} (\vec{v} + \delta\vec{v})^2 - \sum_i \frac{m_i}{2} \vec{v}^2$$

resulting in

$$\Delta E = \sum_i \frac{m_i}{2} (\vec{v} + \delta\vec{v}) \delta\vec{v}$$

because m_i changes at each timestep, ΔE needs to be adjusted.

write $\delta\vec{v} = A\delta\tilde{v}$ with fixed $\delta\tilde{v}$ and adjustable A .

solve quadratic equation to get A :
$$\Delta E = \sum_i \frac{m_i}{2} (A\vec{v}\delta\tilde{v} + A^2\delta\tilde{v}^2)$$

driving turbulence 3

Solve equations of (magneto)hydrodynamics on a computer.

Logarithmic density: $s \equiv \ln \frac{\rho}{\langle \rho \rangle}$

HD equations are then:

$$\frac{\partial s}{\partial t} + (\mathbf{v} \cdot \nabla) s = -\nabla \cdot \mathbf{v} \quad \text{continuity}$$
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -c_s^2 \nabla s + \mathbf{f}, \quad \text{Euler}$$



Leonard Salomon Ornstein (1880-1941)



George Eugene Uhlenbeck (1900 -1988)

stochastic force term that follows
an Ornstein-Uhlenbeck process

named after [Leonard Ornstein](#) and
[George Eugene Uhlenbeck](#)

driving turbulence 4

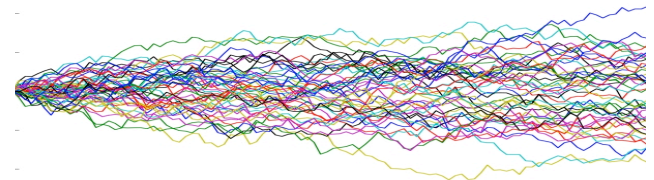
The OU process is a stochastic differential equation describing the evolution of the forcing term in Fourier space (k-space):

$$d\widehat{f}(\mathbf{k}, t) = f_0(\mathbf{k}) \underline{\mathcal{P}}^\zeta(\mathbf{k}) dW(t) - \widehat{f}(\mathbf{k}, t) \frac{dt}{T}$$

the first term on RHS is a diffusion term modeled as a Wiener process $W(t)$ which adds a Gaussian random increment to the vector field given in the previous time step dt .

$$W(t) - W(t - dt) = N(0, dt)$$

where $N(0, dt)$ denotes a Gaussian distribution with zero mean and standard deviation dt



driving turbulence 5

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$$d\widehat{f}(\mathbf{k}, t) = f_0(\mathbf{k}) \underline{\mathcal{P}}^\zeta(\mathbf{k}) dW(t) - \widehat{f}(\mathbf{k}, t) \frac{dt}{T}$$

with projection tensor in Fourier space $\underline{\mathcal{P}}^\zeta(\mathbf{k})$ (Helmholtz decomposition)

in index notation: $\mathcal{P}_{ij}^\zeta(\mathbf{k}) = \zeta \mathcal{P}_{ij}^\perp(\mathbf{k}) + (1 - \zeta) \mathcal{P}_{ij}^\parallel(\mathbf{k}) = \zeta \delta_{ij} + (1 - 2\zeta) \frac{k_i k_j}{|k|^2}$

$\mathcal{P}_{ij}^\perp = \delta_{ij} - k_i k_j / k^2$ fully solenoidal projection operator

$\mathcal{P}_{ij}^\parallel = k_i k_j / k^2$ fully compressive projection operator

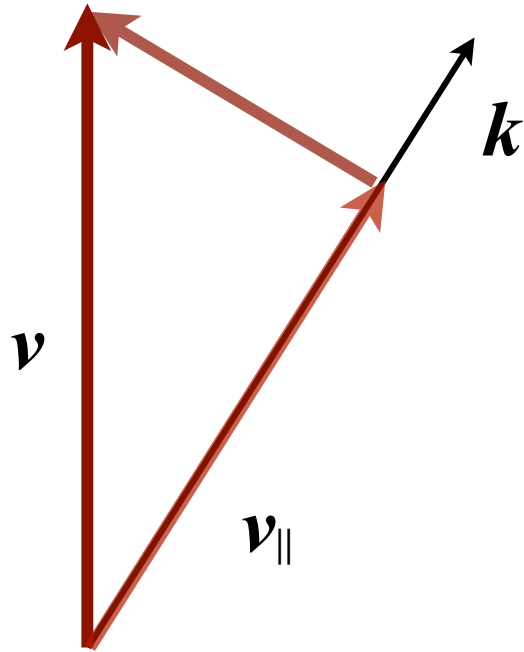


Hermann Ludwig Ferdinand von Helmholtz (1821-1894)

driving turbulence 6

$\mathcal{P}_{ij}^{\perp} = \delta_{ij} - k_i k_j / k^2$ fully solenoidal projection operator

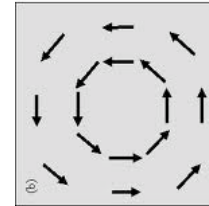
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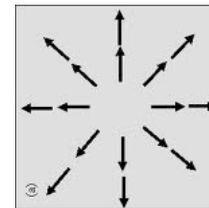
driving turbulence 7

in index notation: $\mathcal{P}_{ij}^{\zeta}(\mathbf{k}) = \zeta \mathcal{P}_{ij}^{\perp}(\mathbf{k}) + (1 - \zeta) \mathcal{P}_{ij}^{\parallel}(\mathbf{k}) = \zeta \delta_{ij} + (1 - 2\zeta) \frac{k_i k_j}{|\mathbf{k}|^2}$

$\zeta = 1$ for purely solenoidal force field



$\zeta = 0$ for purely compressive force field



by adjusting ζ any combination of solenoidal and compressive force fields are possible

driving turbulence 8

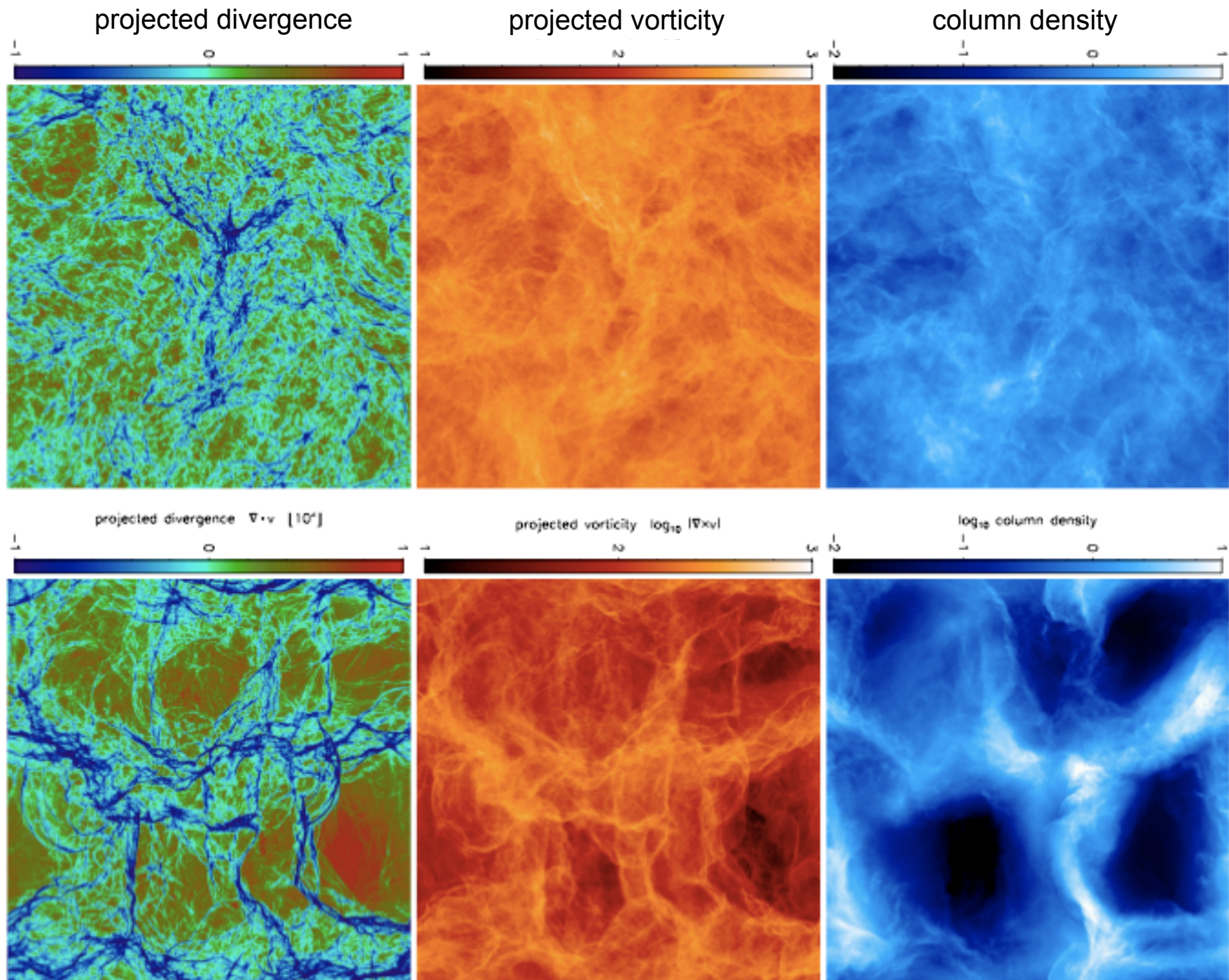
density as function of time / projected density of low-res. 128^3 cube simulation (FLASH)



compressive
larger structures, higher ρ -contrast



rotational
smaller structures, narrow ρ -pdf



driving turbulence 8

The analytical ratio of compressive power to total power can be derived by evaluating the norm of the compressive component of the projection tensor

$$\left| (1 - \zeta) \mathcal{P}_{ij}^{\parallel} \right|^2 = (1 - \zeta)^2,$$

and by evaluating the norm of the full projection tensor

$$\left| \mathcal{P}_{ij}^{\zeta} \right|^2 = 1 - 2\zeta + D\zeta^2,$$

where D denotes the dimensionality of the problem ($D = 1, 2, 3$)

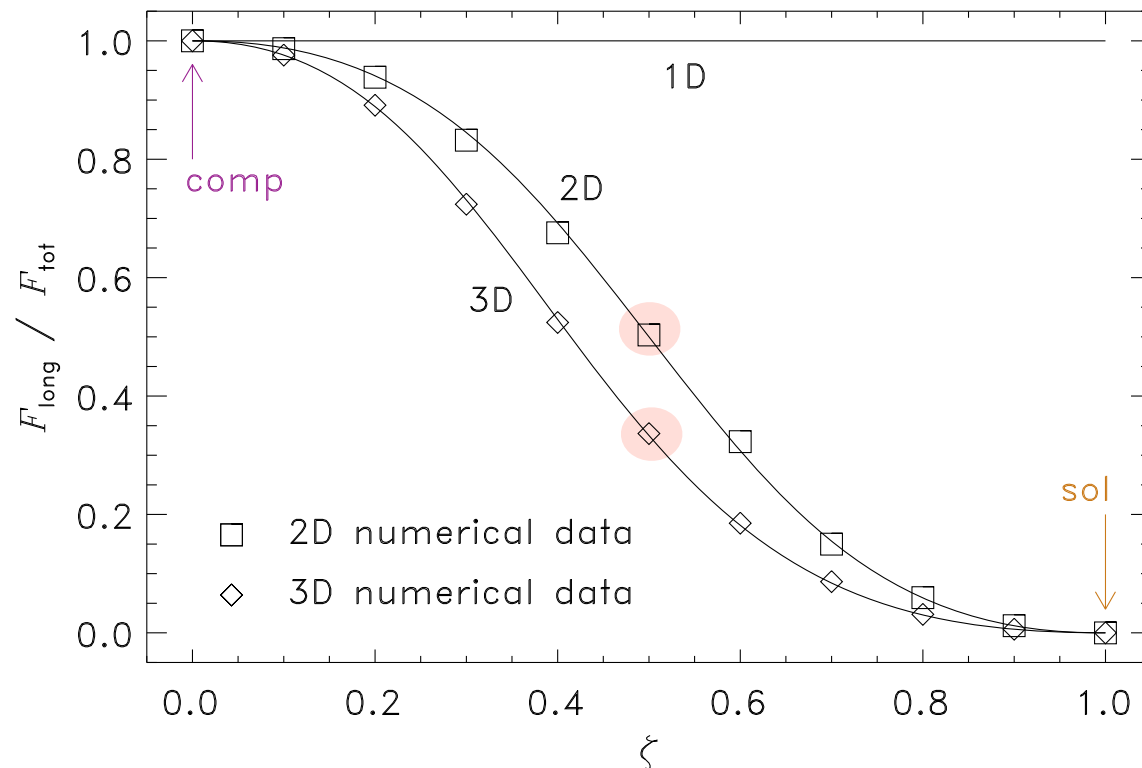
ratio of compressive forcing power to total forcing power is then:

$$\frac{F_{\text{long}}}{F_{\text{tot}}} = \frac{(1 - \zeta)^2}{1 - 2\zeta + D\zeta^2}$$

driving turbulence 9

ratio of compressive forcing power to total forcing power is then:

$$\frac{F_{\text{long}}}{F_{\text{tot}}} = \frac{(1 - \zeta)^2}{1 - 2\zeta + D\zeta^2}$$



natural ratio of solenoidal and compressive forcing for

$$\zeta = 0.5.$$

then we get (for $D=3$)

$$F_{\text{long}} / F_{\text{tot}} = 1/3$$

and ($D=2$)

$$F_{\text{long}} / F_{\text{tot}} = 1/2$$

driving turbulence 10

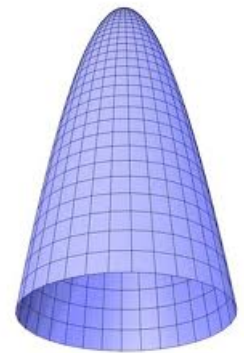
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second term is drift term, and models the exponentially decaying timescale of the force field with itself --> autocorrelation timescale T of the forcing field

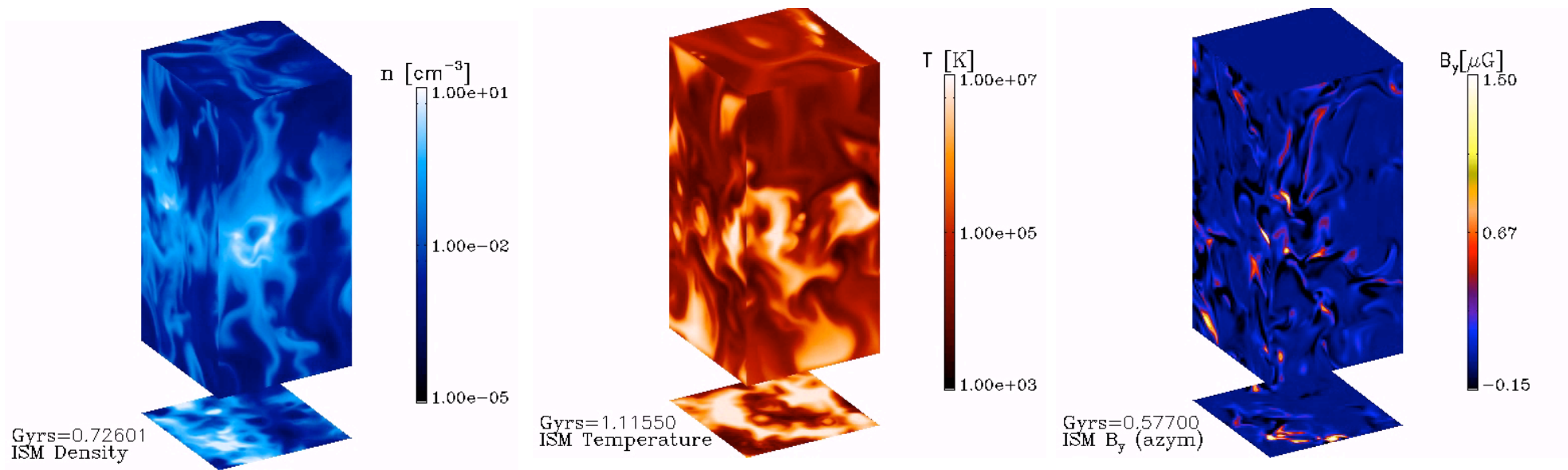
often $T = L/(2V)$ with L being the size of the computational domain and V being the typical crossing time $V = c_s \mathcal{M}$ (with Mach number \mathcal{M} sound speed c_s)

forcing amplitude $f_0(\mathbf{k})$ is a paraboloid in 3D Fourier space, only containing power on the largest scales in a small interval of wavenumbers $k_{\min} < |\mathbf{k}| < k_{\max}$

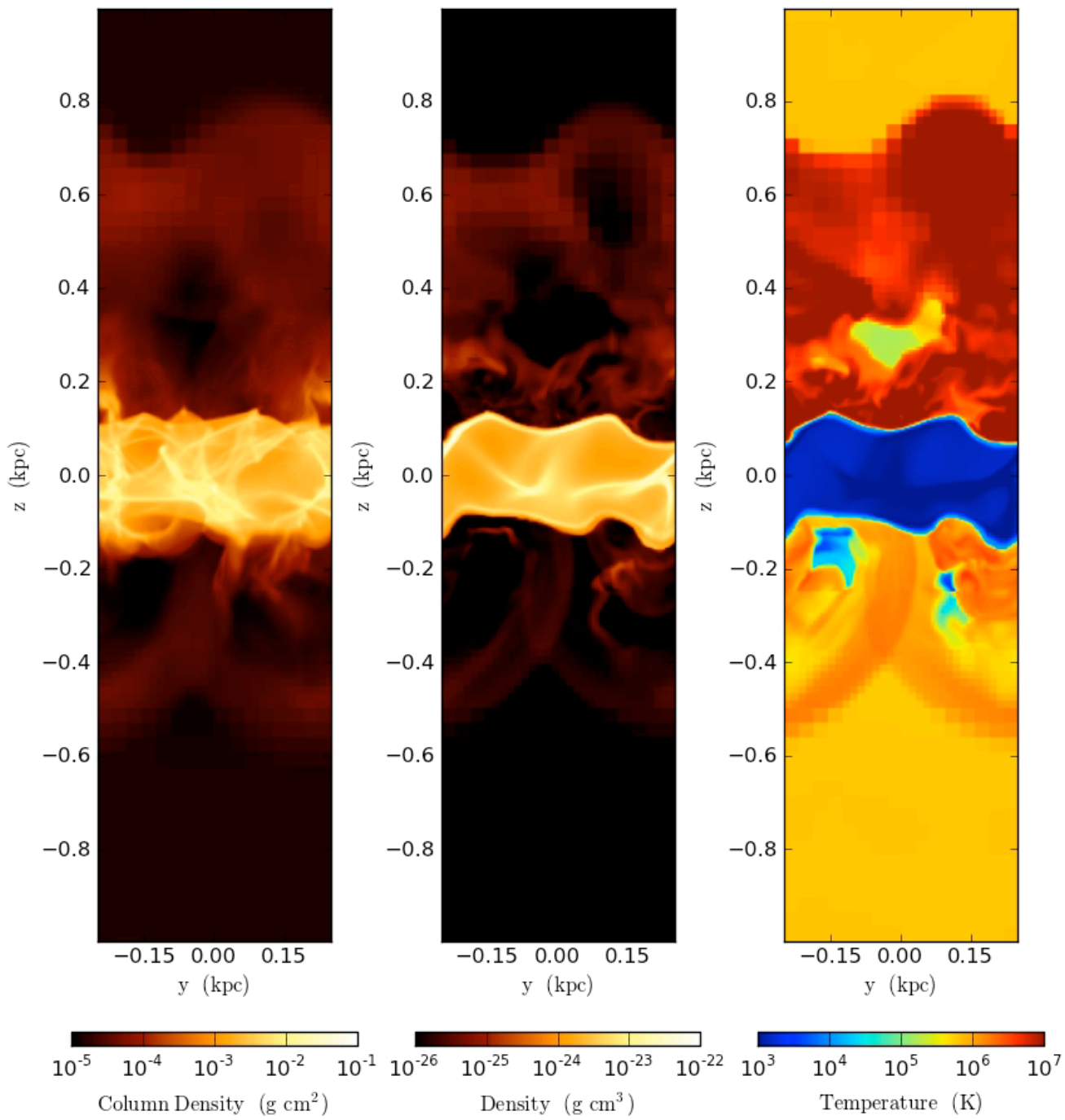


driving turbulence 11

Supernova explosions as drivers of ISM turbulence



movies from Philipp Girichidis (University of Sheffield)



SILCC project

what drives ISM turbulence?

- seems to be driven on large scales, little difference between star-forming and non-SF clouds
→ rules out internal sources
- proposals in the literature
 - supernovae
 - expanding HII regions / stellar winds / outflows
 - spiral density waves
 - magneto-rotational instability
 - more recent idea: accretion onto disk

what drives ISM turbulence?

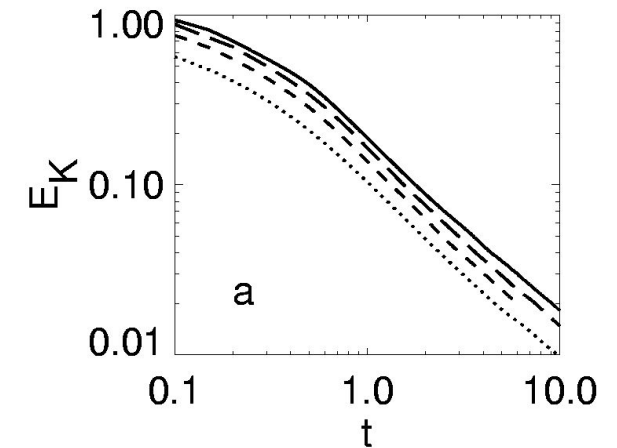
some energetic arguments...

energy decay by turbulent dissipation:

$$\begin{aligned}\dot{e} &\simeq -(1/2)\rho v_{\text{rms}}^3/L_d \\ &= -(3 \times 10^{-27}) \text{ erg cm}^{-3} \text{ s}^{-1} \left(\frac{n}{1 \text{ cm}^{-3}} \right) \\ &\quad \times \left(\frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right)^3 \left(\frac{L_d}{100 \text{ pc}} \right)^{-1},\end{aligned}$$

decay timescale:

$$\begin{aligned}\tau_d = e/\dot{e} &\simeq L_d/v_{\text{rms}} \\ &= (9.8 \text{ Myr}) \left(\frac{L_d}{100 \text{ pc}} \right) \left(\frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right)^{-1},\end{aligned}$$



(Mac Low et al. 1999)

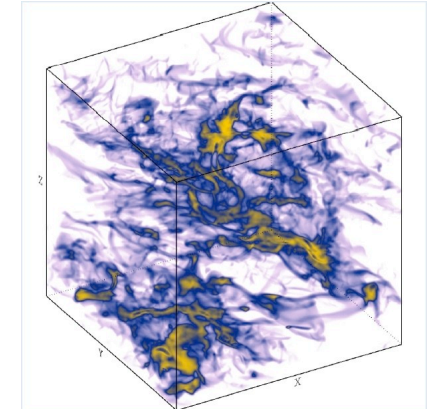
what drives ISM turbulence?

magneto-rotational instability:

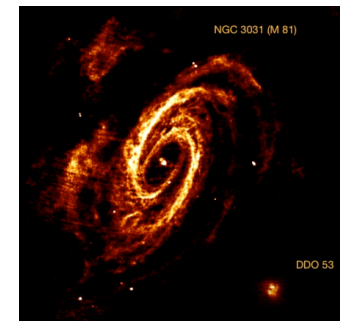
$$\dot{e} = (3 \times 10^{-29} \text{ erg cm}^{-3} \text{ s}^{-1}) \left(\frac{B}{3 \mu\text{G}} \right)^2 \left(\frac{\Omega}{(220 \text{ Myr})^{-1}} \right).$$

gravitational instability (spiral waves)

$$\begin{aligned} \dot{e} &\simeq G(\Sigma_g/H)^2 \lambda^2 \Omega \\ &\simeq (4 \times 10^{-29} \text{ erg cm}^{-3} \text{ s}^{-1}) \\ &\quad \times \left(\frac{\Sigma_g}{10 M_\odot \text{ pc}^{-2}} \right)^2 \left(\frac{H}{100 \text{ pc}} \right)^{-2} \\ &\quad \times \left(\frac{\lambda}{100 \text{ pc}} \right)^2 \left(\frac{\Omega}{(220 \text{ Myr})^{-1}} \right), \end{aligned}$$



(from Pietek & Ostriker 2005)



(from Walter et al. 2008)

what drives ISM turbulence?

protostellar outflows

$$\begin{aligned} \dot{e} &= \frac{1}{2} f_w \eta_w \frac{\dot{\Sigma}_*}{H} v_w^2 \\ &\simeq (2 \times 10^{-28}) \text{ erg cm}^{-3} \text{ s}^{-1} \left(\frac{H}{200 \text{ pc}} \right)^{-1} \left(\frac{f_w}{0.4} \right) \\ &\quad \times \left(\frac{v_w}{200 \text{ km s}^{-1}} \right) \left(\frac{v_{\text{rms}}}{10 \text{ km s}^{-1}} \right) \\ &\quad \times \left(\frac{\dot{\Sigma}_*}{4.5 \times 10^{-9} M_\odot \text{ pc}^{-2} \text{ yr}^{-1}} \right), \end{aligned}$$

(Li & Nakamura 2006, Wang et al. 2010 vs. Banerjee et al. 2008)

expanding HII regions

$$\begin{aligned} \dot{e} &= \frac{\langle \delta p \rangle \mathcal{N}(>1) v_i}{V t_i} \\ &= (3 \times 10^{-30}) \text{ erg s}^{-3} \\ &\quad \times \left(\frac{N_H}{1.5 \times 10^{22} \text{ cm}^{-2}} \right)^{-3/14} \left(\frac{M_{cl}}{10^6 M_\odot} \right)^{1/14} \\ &\quad \times \left(\frac{\langle M_* \rangle}{440 M_\odot} \right) \left(\frac{\mathcal{N}(>1)}{650} \right) \left(\frac{v_i}{10 \text{ km s}^{-1}} \right) \\ &\quad \times \left(\frac{H_c}{100 \text{ pc}} \right)^{-1} \left(\frac{R_{sf}}{15 \text{ kpc}} \right)^{-2} \left(\frac{t_i}{18.5 \text{ Myr}} \right)^{-1} \end{aligned}$$

(note: different numbers by Matzner 2002)

(from Mac Low & Klessen, 2004)

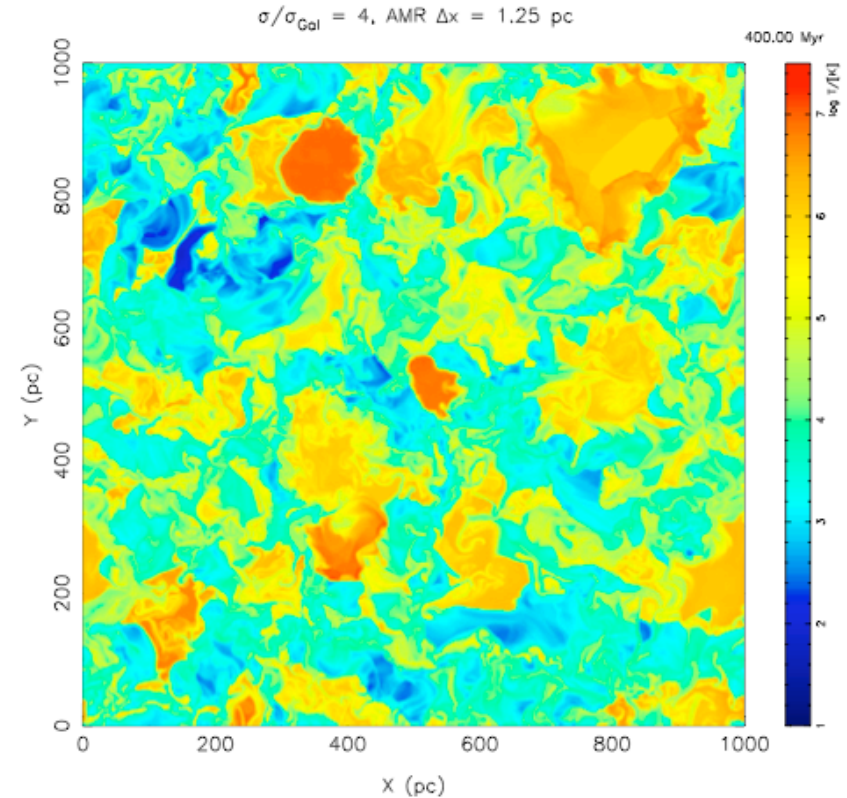
what drives ISM turbulence?

supernovae

$$\begin{aligned}\dot{e} &= \frac{\sigma_{SN} \eta_{SN} E_{SN}}{\pi R_{sf}^2 H_c} \\ &= (3 \times 10^{-26} \text{ erg s}^{-1} \text{ cm}^{-3}) \left(\frac{\eta_{SN}}{0.1} \right) \left(\frac{\sigma_{SN}}{1 \text{ SNU}} \right) \\ &\quad \times \left(\frac{H_c}{100 \text{ pc}} \right)^{-1} \left(\frac{R_{sf}}{15 \text{ kpc}} \right)^{-2} \left(\frac{E_{SN}}{10^{51} \text{ erg}} \right).\end{aligned}$$

in star-forming parts of the disk,
clearly SN provide enough energy
to compensate for the decay of
ISM turbulence.

BUT: what is outside the disk?



(distribution of temperature in SN driven disk turbulence, by
de Avillez & Breitschwerdt 2004)

accretion driven turbulence

- yet another thought:
 - astrophysical objects *form* by *accretion* of ambient material
 - the *kinetic energy* associated with this process is a key agent *driving internal turbulence*.
 - this works on *ALL* scales:
 - galaxies
 - molecular clouds
 - protostellar accretion disks

concept

- turbulence decays on a crossing time

$$\tau_d \approx \frac{L_d}{\sigma}$$

- energy decay rate

$$\dot{E}_{\text{decay}} \approx \frac{E}{\tau_d} = -\frac{1}{2} \frac{M\sigma^3}{L_d}$$

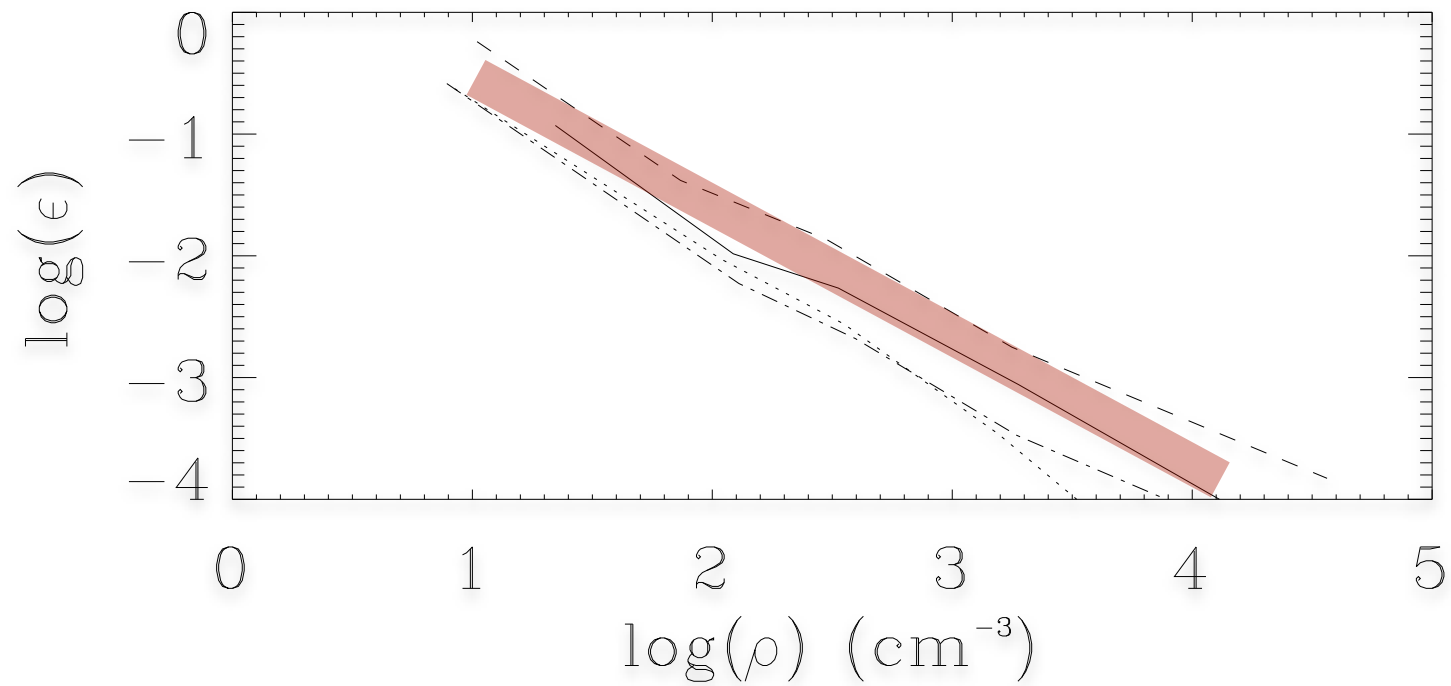
- kinetic energy of infalling material

$$\dot{E}_{\text{in}} = \frac{1}{2} \dot{M}_{\text{in}} v_{\text{in}}^2$$

- can both values match, modulo some efficiency?

$$\epsilon = \left| \frac{\dot{E}_{\text{decay}}}{\dot{E}_{\text{in}}} \right|$$

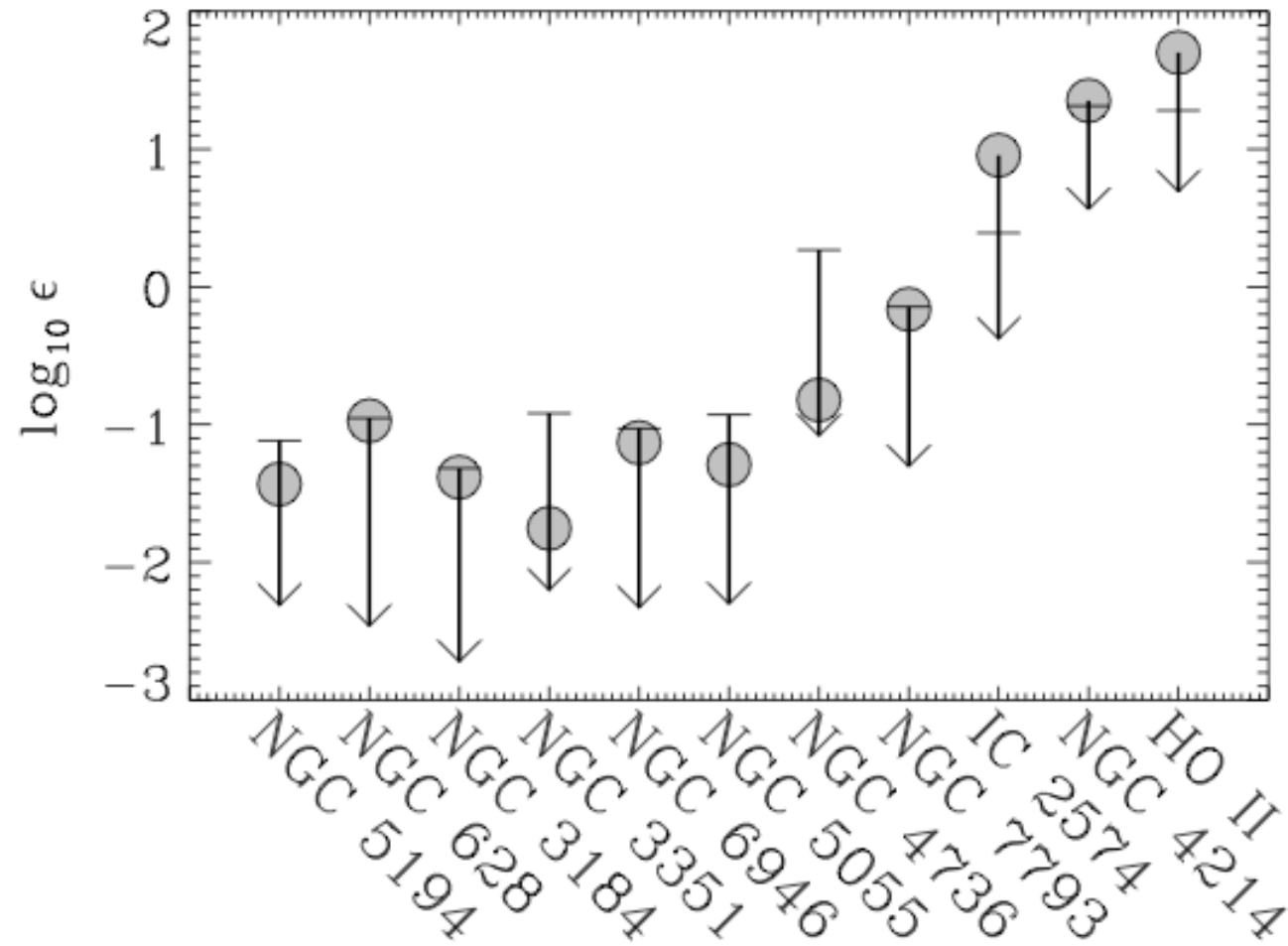
some estimates from convergent flow studies



application to galaxies

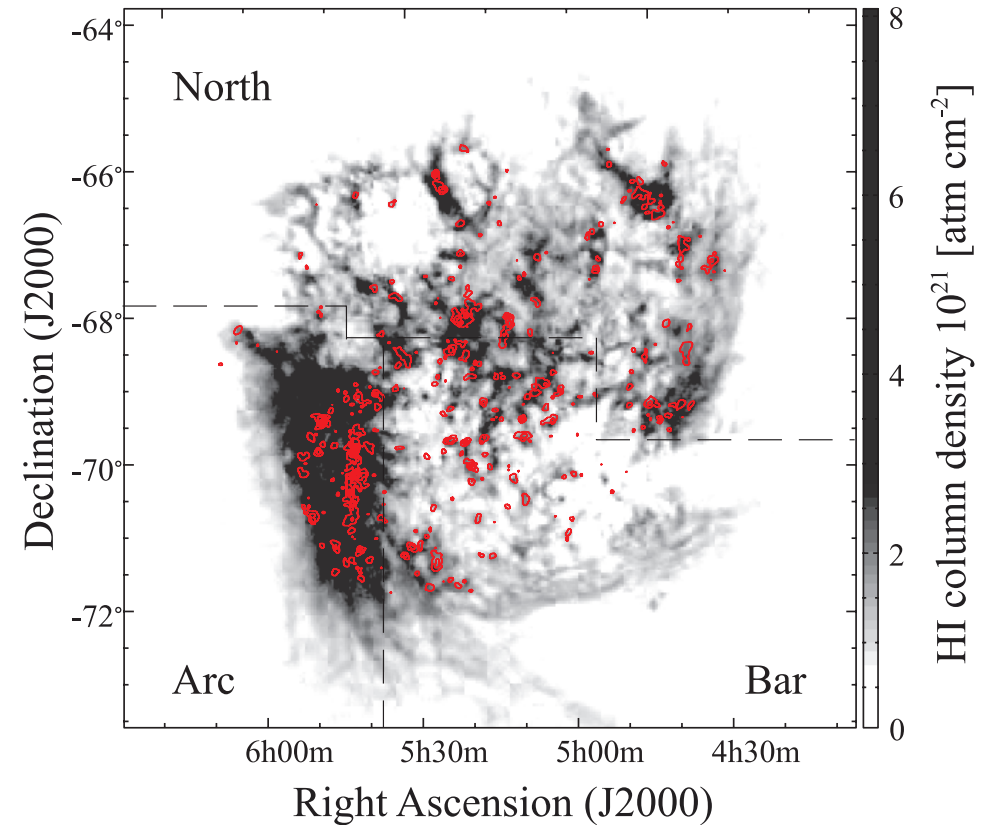
- underlying assumption
 - galaxy is in steady state
 - > accretion rate equals star formation rate
 - what is the required efficiency for the method to work?
- study Milky Way and 11 THINGS
 - excellent observational data in HI:
velocity dispersion, column density, rotation curve

11 THINGS galaxies



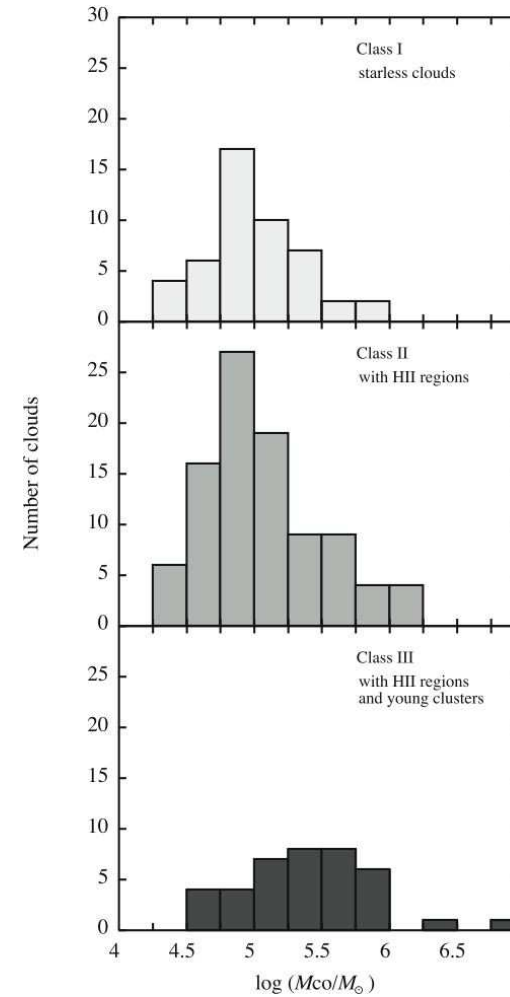
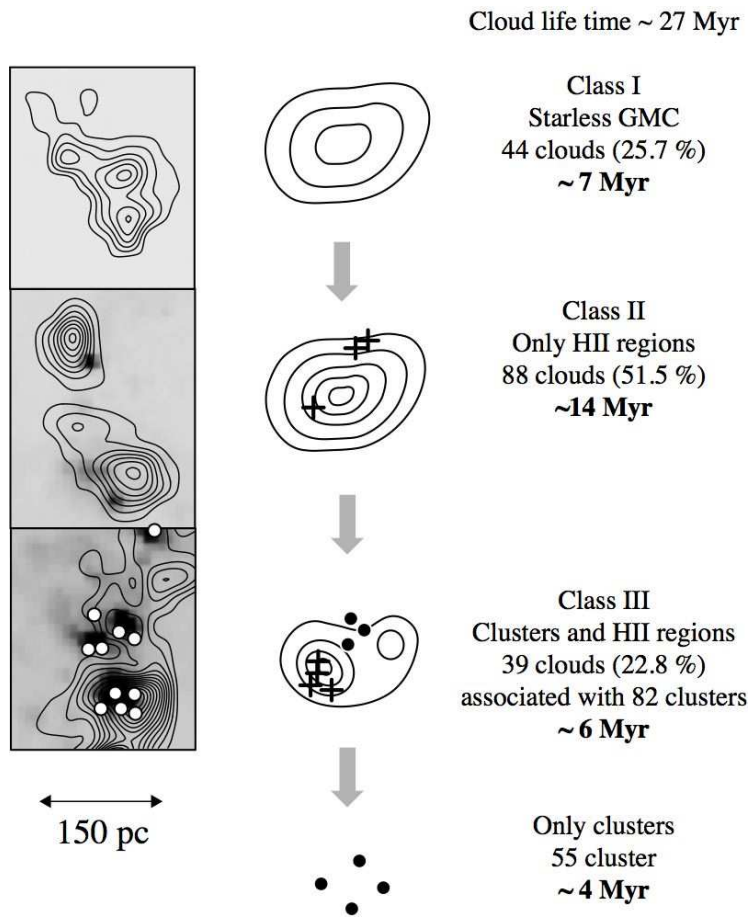
molecular cloud scales

- molecular clouds grow in mass
- this is inferred by looking at molecular clouds in different evolutionary phases in the LMC (Fukui et al. 2008, 2009)



Fukui et al. (2009)

molecular cloud scales



Blitz et al. (2007, PPV)

some further thoughts

- method works for Milky Way type galaxies:
 - required efficiencies are $\sim 1\%$ only!
- relevant for outer disks (extended HI disks)
 - there are not other sources of turbulence (certainly not stellar sources, maybe MRI)
- works well for molecular clouds
 - example clouds in the LMC (Fukui et al.)
- potentially interesting for TTS
 - model reproduces $dM/dt - M$ relation (e.g Natta et al. 2006, Muzerolle et al. 2005, Muhanty et al. 2005, Calvet et al. 2004, etc.)

end

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Hokusai: In the wake of the great wave of Tanakawa